Problem 7.52

For the potential energy well:

a.) Determine (and justify) the sign of the associated force field at each identified point:

Justification: According to

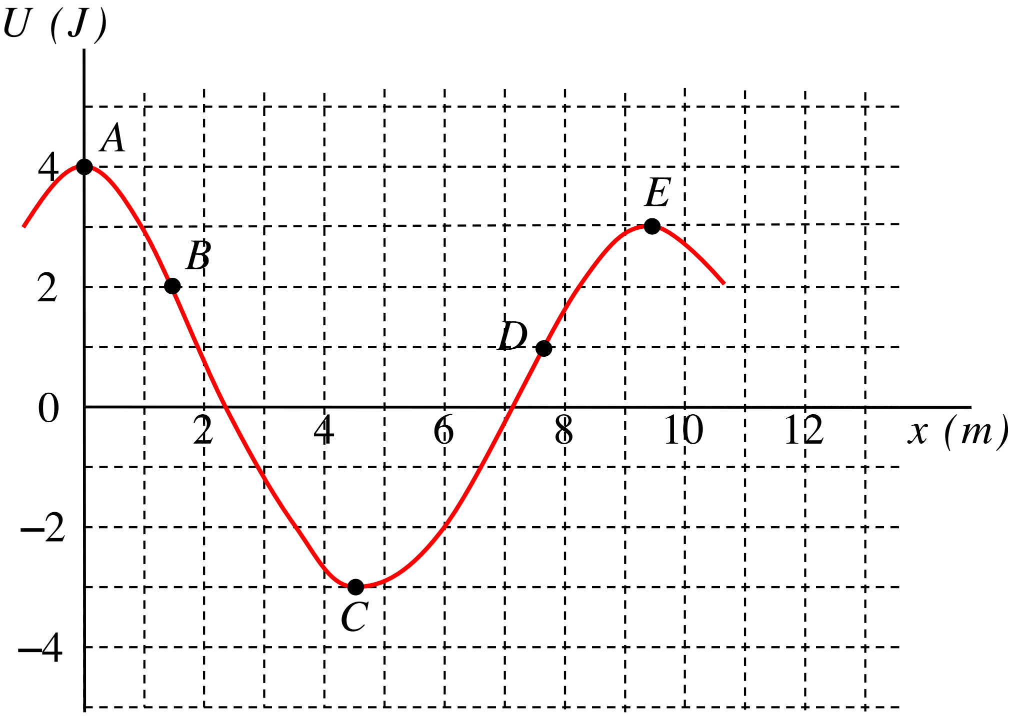
$$\vec{F} = -\vec{\nabla}U = -\frac{dU}{dx}\hat{i}$$

minus the slope of the potential energy graph is equal to the force, sign included. As such:

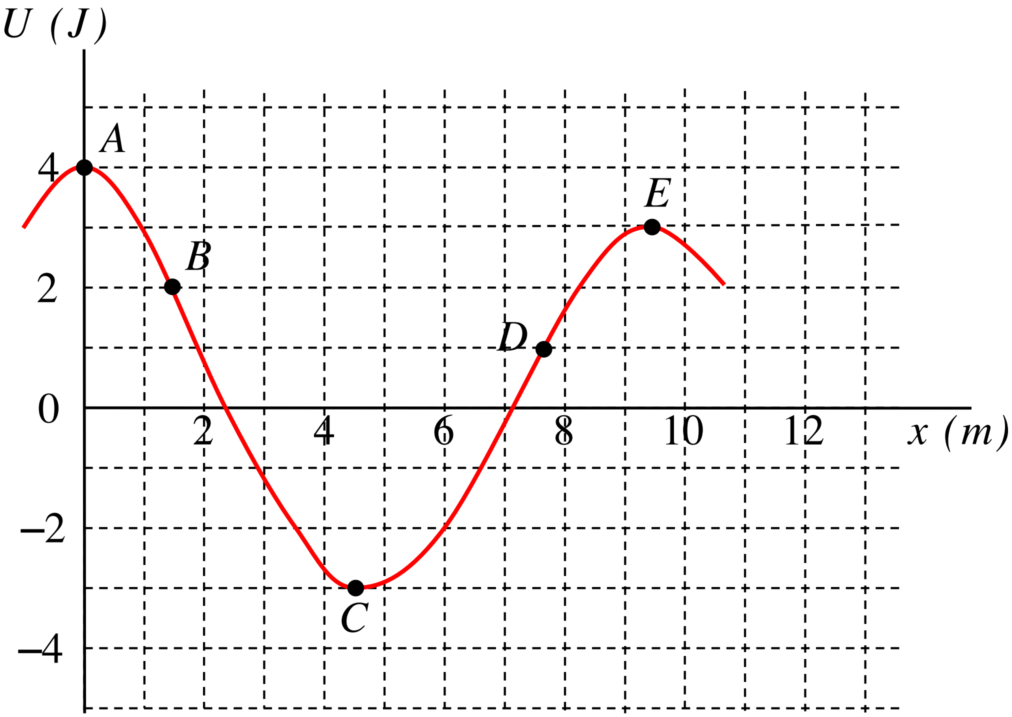
The force is zero at points “A,” “C” and “E.”

The force is positive at point “B.”

The force is negative at point “D.”



b.) Where are there equilibrium points, and what kind are they?



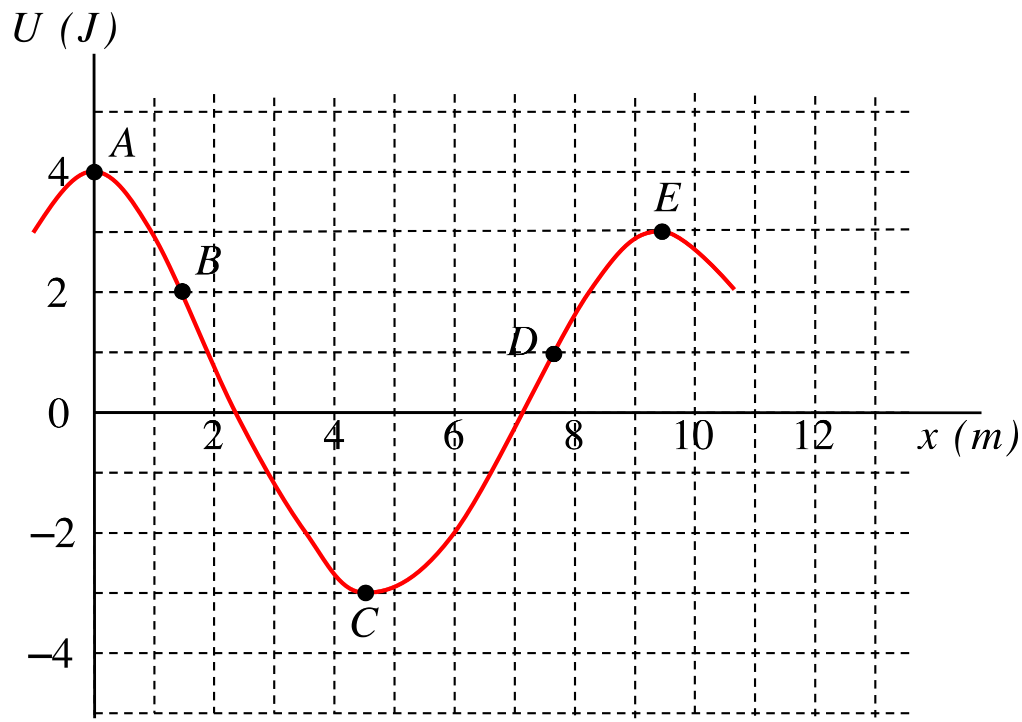
An equilibrium point is a point where, if a body in the field is released, it will remain stationary. *Stable equilibrium* means that if the body is displaced from the point slightly, it will return to the point (i.e., the body is in a valley). These points are at a *minimum potential* energy in a potential energy well. *Point C* is a stable equilibrium point (think about it, if you displace a body at *C*, it will end up with *more* potential energy that it had—when released, it will convert that potential energy into motion returning the body to the lower potential energy state, which is to say, back to *Point C*). *Unstable equilibrium* means that if the body is displaced from the point slightly, it will accelerate away from the point (the top of a hill). *Points A and E* are unstable equilibrium points as moving from those points slightly allows the body to decrease its potential energy. You would have to feed energy into the system to get the body to return to those points.

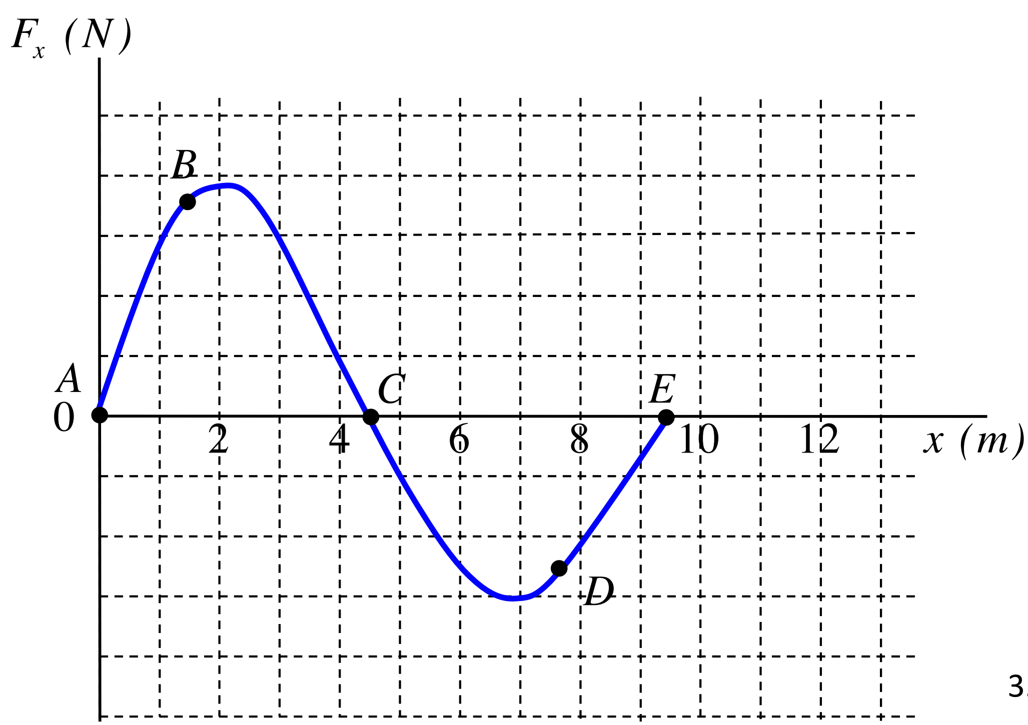
c.) Determine the *force versus position* graph for this potential energy function.

Using once again the relationship

$$\vec{F} = -\vec{\nabla}U = -\frac{dU}{dx}\hat{i},$$

a graph of *minus* the slope of our curve will do the trick. That is shown below and to the right.





Note that this is very much an approximation—there are areas of the graph above where it looks as though the slope is close to constant, which would suggest horizontal sections in the force graph, but that was a little more nuanced than I got—sorry!